Candidate evidence

Question 3

Response 1

ENTER NUMBER OF QUESTION

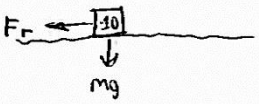
3.

$m_1u_1 + m_2u_2 = m_tv$

$10u_1 + 5 \times 0 = (10+5)v$

$10u_1 = 15v$

$v = \frac{2}{3}u_1$



$F_r = \mu R$

$= \mu mg$

$= 0.25 \times 10 \times 9.8$

$= 24.5\,N$

$E_w = Fs$

$= 24.5 \times 20$

$= 490\,J$

By conservation of energy:

$E_k = E_w$

$\frac{1}{2}mu_1^2 = 490$

$980 = 10u_1^2$

$u_1^2 = 98$

$u_1 = 7\sqrt{2} = 9.899\,ms^{-1}$

$v = \frac{2}{3}u_1$

$= \frac{2}{3}(9.899)$

$= 6.6\,ms^{-1}$

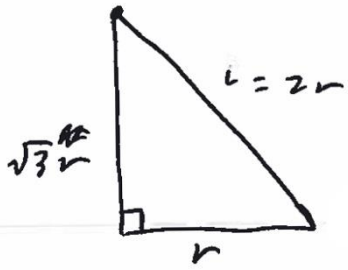
Question 5

Response 2

ENTER NUMBER OF QUESTION

5

$L = 2r$



$(2r)^2 - r^2$

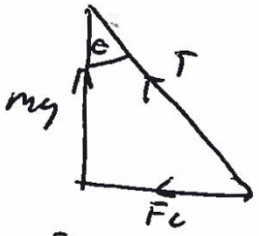
$= 4r^2 - r^2$

$= 3r^2$

$\sqrt{3}r$

$F_c = mr\omega^2 = mr\omega^2$

$a = r\omega^2 = m\omega^2 \frac{L}{2}$



$\omega^2 = \frac{F_c}{mr}$

$\tan\theta = \frac{F_c}{mg} = \frac{m\omega^2 r}{mg}$

$T^2 = (mg)^2 + F_c^2$

$T = m^2g^2 + m^2\omega^4 r^2$

$\sqrt{3}r = g$

$r = \frac{g}{\sqrt{3}}$

$a_c = \frac{g}{\sqrt{3}}$

$r = \frac{L}{2}$

$\omega^2 = \frac{a}{r} = \frac{g}{\sqrt{3}} \div \frac{L}{2}$

$\omega^2 = \frac{2g}{\sqrt{3}L}$

Question 7

Response 3

ENTER NUMBER OF QUESTION

7a $a = 6 \quad T = 10 \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{10} = \frac{1}{5}\pi.$

$x = a\sin\omega t \Rightarrow 4 = 6\sin\frac{\pi}{5}t$

$\therefore t = 60.79s,\ 121.59s.$

~~b) $t = 60.79: \ V^2 = \omega^2(a^2 - x^2)$~~

~~$V^2 = \frac{\pi^2}{25}(36 - 16) \Rightarrow V^2 = 7.896$~~

~~$\therefore V = 2.81 ms^{-1}$~~

~~$t = 121.59: \ V^2 = \omega^2(a^2 - x^2)$~~

~~$V^2 = \frac{\pi^2}{25}$~~

7b $V^2 = \omega^2(a^2 - x^2)$

$V^2 = \frac{\pi^2}{25}(36 - 16) = 7.896$

$\therefore V = 2.81 ms^{-1}.$

$\Rightarrow V(121.59s) = -2.81 ms^{-1}.$

V is negative since particle is travelling back from max. displacement to the equilibrium position.

Response 4

ENTER NUMBER OF QUESTION

7a)

$A = 6m$

$T = 10s$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{10} = \frac{\pi}{5}$ rads^{-1}

$x = A\sin\omega t$

$4 = 6\sin\frac{\pi}{5}t$

$t = 1.16s$

$6 = 6\sin\frac{\pi}{5}t$

$t = \frac{5}{2}s$

$\frac{5}{4} + 1.16 =$ 2.41s

7b)

$v^2 = \omega^2(A^2 - x^2)$

$= \left(\frac{\pi}{5}\right)^2(6^2 - 4^2)$

$= 7.89568$

$v = \pm 2.8 ms^{-1}$ They are in opposite direction.

Question 8

Response 5

ENTER NUMBER OF QUESTION

8. $x = t(t+4)$ $\quad y = t(1-t)^3$

$\frac{y'(t)}{x'(t)}$

$x'(t) = t^2 + 4t$ $\quad y'(t) = 3t(1-t)^2$

$= 2t + 4$

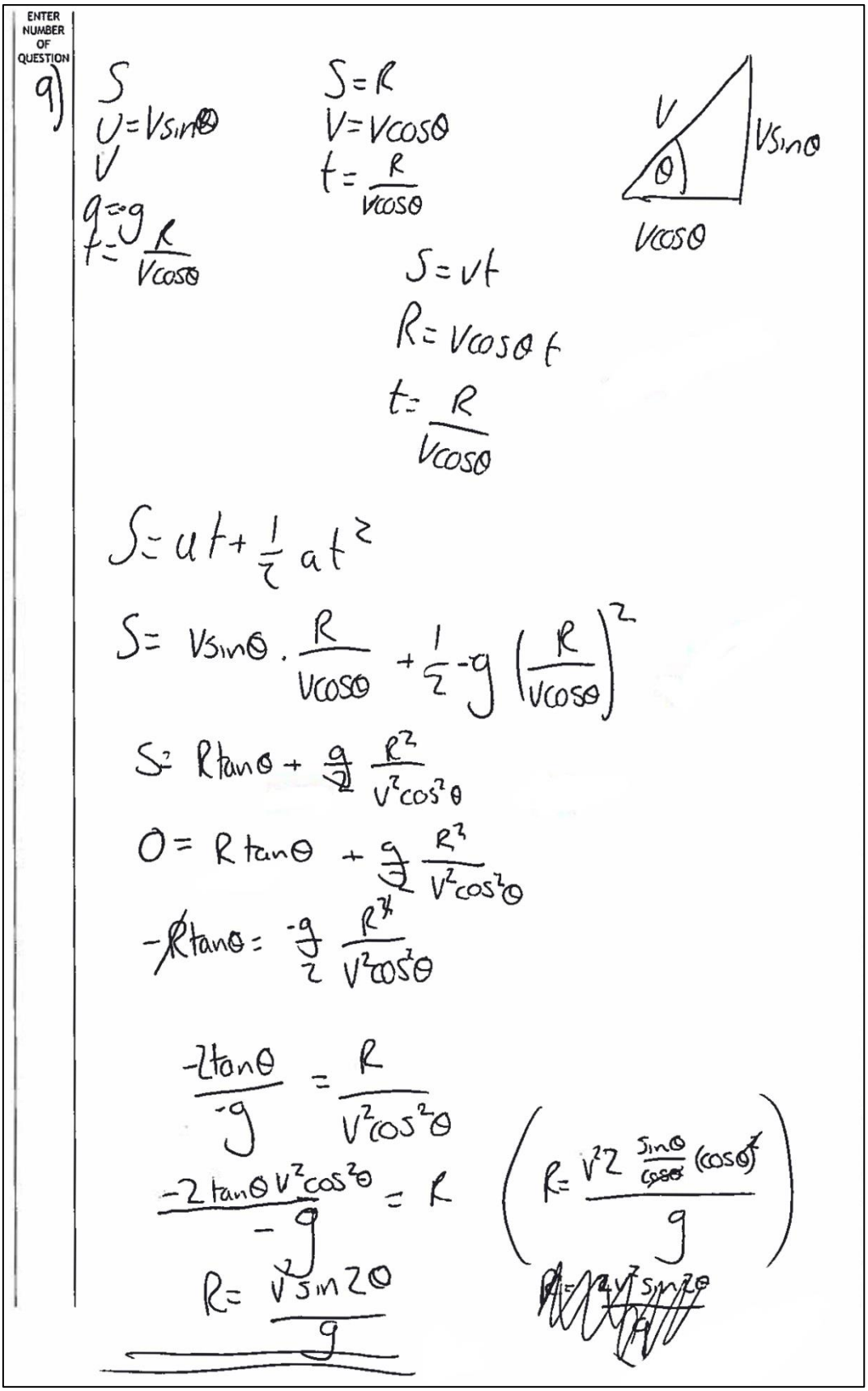
$= \frac{3t(1-t)^2}{2t+4}$

$= \frac{3(3)(1-(3))^2}{2(3)+4}$

$= 3.6 \text{ m/s}$

Question 9

Response 6



ENTER NUMBER OF QUESTION

b)

$$\frac{v^2 \sin 2(30)}{g} = \frac{v^2 \sin 2(35)}{g} - 5$$

$$v^2 \sin 60 = v^2 \sin 70 - 5g$$

$$v^2 \sin 60 - v^2 \sin 70 = -5g$$

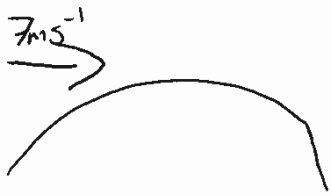
$$v^2 \sin 70 - v^2 \sin 60 = 5g$$

$$v^2 (\sin 70 - \sin 60) = 5g$$

$$v^2 = \frac{5g}{(\sin 70 - \sin 60)}$$

$$v = 25.8 \text{ms}^{-1}$$

ENTER NUMBER OF QUESTION



$$R+5 = \frac{V^2 \sin 2(35)}{g} + 7$$

$$R = \frac{(25.8)^2 \sin 70}{g} + 2$$

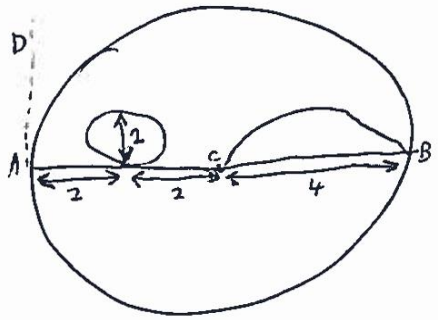
$$R = 65.8\text{m}$$

Question 10

Response 7

ENTER NUMBER OF QUESTION

10(a)



$A = \pi r^2$

$= 16\pi$

	Mass	CoM from AD	CoM from AB
whole Circle	$16\pi m$	4	0
Circle hole	$-\pi m$	2	1
Semi-circle hole	$-4\pi m$	6	$\frac{8}{3}\pi$
lamina	$11\pi m$	$\bar{x}$	$\bar{y}$

$\bar{y}$ of semi-circle hole $= \frac{4r}{3\pi}$ up from AB

$= \frac{4|2|}{3\pi} = \frac{8}{3}\pi$

$11\pi m\,\bar{x} = 16\pi m - 2\pi m - 24\pi m$

$11\bar{x} = 38$

$\bar{x} = \frac{38}{11} = 3.45$

ENTER NUMBER OF QUESTION

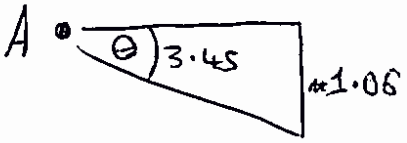
$11\pi m\bar{y} = -\pi m - \frac{32}{3}\pi m$

$11\bar{y} = -\frac{35}{3}$

$\bar{y} = -\frac{35}{33} = -1.06$

$CoM(3.45, -1.06)$

(b)



$\tan\theta = \frac{1.06}{3.45}$

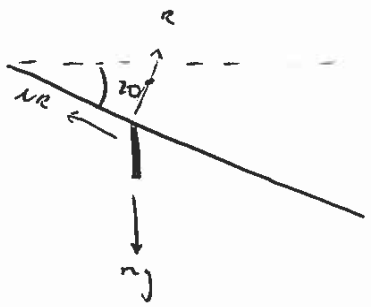
$\theta = 17.1°$

Question 12

Response 8

ENTER NUMBER OF QUESTION

12a)



~~R = nR~~ $R = mg\cos\theta$

$F = ma$

$ma = mg\sin\theta - nR$

$ma = mg\sin\theta - n\,mg\cos\theta$

$a = g\sin\theta - ng\cos\theta \qquad \theta = 20°$

$= 0.5691\ldots$

$= 0.6\,ms^{-2}$

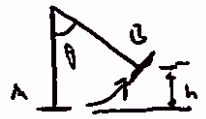
~~S.U.V.A~~ $v^2 = u^2 + 2as$

$v = \sqrt{2^2 + 2(0.6)(20)}$

$v = 2\sqrt{7}\,ms^{-1}$

ENTER NUMBER OF QUESTION

12b)



E @ A : $E_K + E_P$: $\frac{1}{2}m(2\sqrt{7})^2$

E @ B : $E_K + E_P$: mgh

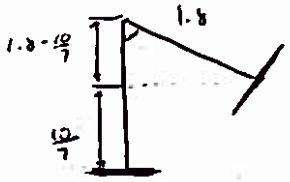
by the principle of conservation of energy :

E @ A : E @ B

$\frac{1}{2}m(2\sqrt{7})^2 = mgh$

$\frac{\frac{1}{2}(2\sqrt{7})^2}{g} = h$

$h = \frac{10}{7}$ m



(a) $\theta = \frac{1.8 - \frac{10}{7}}{1.8}$

$\theta = \cos^{-1}\left(\frac{1.8 - \frac{10}{7}}{1.8}\right)$

$= 78.0513...$

$\theta = 78.1°$

Question 13

Response 9

ENTER NUMBER OF QUESTION

13) $\int_0^{\sqrt{5}} \frac{2x^3}{\sqrt{x^2+4}}\,dx$

$u = x^2+4$

$\frac{du}{dx} = 2x$

$dx = \frac{1}{2x}\,du$

$\Rightarrow \int_0^{\sqrt{5}} \frac{2x^3}{u} \times \frac{1}{2x}\,du$

$u = x^2+4$

$u-4 = x^2$

$\sqrt{u-4} = x$

$\int_0^{\sqrt{5}} \frac{x^2}{u}\,du$

$\Rightarrow \int_0^{\sqrt{5}} \frac{u-4}{u}\,du = \int_0^{\sqrt{5}} 1 - \frac{4}{u}\,du$

$= \int_0^{\sqrt{5}} 1 - 4\int_0^{\sqrt{5}} \frac{1}{u}\,du$

$= |u|_0^{\sqrt{5}} - 4\left|\ln|u|\right|_0^{\sqrt{5}}$

$\Rightarrow |x^2+4|_0^{\sqrt{5}} - 4\left|\ln|x^2+4|\right|_0^{\sqrt{5}}$

$= 5 - 4\left|\frac{\ln|9|}{\ln|4|}\right|$

$= 5 - 4\ln\left|\frac{9}{4}\right|$

$= 1.756$

Question 14

Response 10

ENTER NUMBER OF QUESTION

14)

$m = 70$
$h = 40$
$l = 10$
$\lambda = 1000$

$T = \lambda \frac{x}{l}$

$= \frac{1000}{10} x$

$= 100x$

At equilibrium

$70g = 100x$

$x = \frac{7g}{10}$

at 10m

$v^2 = u^2 + 2as$

$= 2 \times 9.8 \times 10$

$v = 14 ms^{-1}$

$F = mg - \lambda \frac{x}{l}$

$= 70g - 100x$

$a = g - \frac{10}{7}x$

$v \frac{dv}{dx} = g - \frac{10}{7}x$

$\int v\,dv = \int g - \frac{10}{7}x$

$\frac{v^2}{2} + c = gx - \frac{5}{7}x^2$

when $x = 0$, $v = 14$

$c = -98$

$\frac{v^2}{2} = gx - \frac{5}{7}x^2 + 98$

when $v = 0$

$gx - \frac{5}{7}x^2 + 98 = 0$

$5x^2 - 7gx - 686 = 0$

$x = \frac{7g \pm \sqrt{18425.96}}{10}$

$x = 20.43$ or -3.3

$x > 0 \Rightarrow x = 20.43$

ENTER NUMBER OF QUESTION

14)

(cont.)

$40 - 10 - 20 \cdot 43$

$h = 9 \cdot 57 \text{ m}$

Question 15

Response 11

ENTER NUMBER OF QUESTION

15) a)

$$\frac{d^2x}{dt^2} + 0.4\frac{dx}{dt} + 0.04x = 0$$

$$x^2 + 0.4x + 0.04 = 0$$

$$x = \frac{-0.4 \pm \sqrt{0.4^2 - 4 \times 1 \times 0.04}}{2}$$

$x = -0.2 \quad x = -0.2$

~~$x = Ae^{-0.2t} + Be^{-0.2t}$~~ $x = e^{-0.2t}(A+Bt)$

$\Rightarrow$ ~~$\frac{dx}{dt} = -0.2Ae^{-0.2t} - 0.2Be^{-0.2t}$~~ $\frac{dx}{dt} = -0.2e^{-0.2t}(A+B)$

When $t=0$, $x=1.5$, $\frac{dx}{dt} = 0.5$

$\Rightarrow$ ~~$1.5 = A + B$~~ ~~$0.5 = -0.2(A+B)$~~ ~~$\Rightarrow 0.5 = -0.2(1.5+B)$~~

$\Rightarrow 1.5 = A$

$0.5 = A + B$

$0.5 = 1.5 + B$

$-1 = B$

$\Rightarrow x = e^{-0.2t}(1.5+t)$

ENTER NUMBER OF QUESTION

15

b) $x = e^{-0.2t}(1.5+t)$

$\Rightarrow x = e^{-0.2(2)}(1.5+2)$

$x = 2\,m$

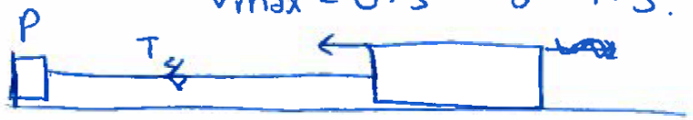
Response 12

ENTER NUMBER OF QUESTION

15.

a)

$V_{max} = 0.5 \quad a = 1.5$.



$x = a\cos\omega t$.

$$\frac{d^2x}{dt^2} + 0.4\frac{dx}{dt} + 0.04x = 0$$

Aux equn: $m^2 + 0.4m + 0.04 = 0$.

$$(m + 0.2)(m + 0.2) = 0$$

$m = -0.2$. $\quad m + 0.2 = 0$

$\Rightarrow m = -0.2$.

$$x = Ae^{-0.2t} + Bte^{-0.2t}$$

when $t = 0$, $x = 1.5$.

$1.5 = A$

$A = \frac{3}{2}$.

$$x = \frac{3}{2}e^{-0.2t} + Bte^{-0.2t}$$

$$\frac{dx}{dt} = -\frac{3}{10}e^{-0.2t} + Be^{-0.2t} - 0.2Bte^{-0.2t}$$

when $t = 0$, $\frac{dx}{dt} = 0.5$

$$\frac{1}{2} = -\frac{3}{10} + B$$

$$B = \frac{1}{2} + \frac{3}{10}$$

$$= \frac{4}{5}$$

$$\Rightarrow x = \frac{3}{2}e^{-0.2t} + \frac{4}{5}te^{-0.2t}.$$

ENTER NUMBER OF QUESTION

b) when $t=2$

$$x=\left(\frac{3}{2}\times e^{-0.2\times 2}\right)+\left(\frac{4}{5}\times 2\times e^{-0.2\times 2}\right)$$

$$=2.1\text{m}.$$

Question 16(a)

Response 13