Candidate 1 evidence

QUESTION NUMBER

3.(a)

$\ddot{y} = -g$

$\dot{y} = U\sin\theta - gt$

$y = U\sin\theta t - \frac{1}{2}gt^2$

When $\dot{y} = 0$, $h = H$

$0 = U\sin\theta - gt$

$t = \frac{U}{g}\sin\theta$

$H = \frac{U^2\sin^2\theta}{g} - \frac{1}{2}g \times \left(\frac{U\sin\theta}{g}\right)^2$

$= \frac{U^2\sin^2\theta}{g} - \frac{g}{2} \times \frac{U^2\sin^2\theta}{g^2}$

$= \frac{U^2\sin^2\theta}{2g}$

3.(b)

$H = \frac{40^2\sin^2 27^\circ}{2\times 9.8}$

$50 - H = 33.2\text{m}$ (1dp)

QUESTION NUMBER

15.

$mU = (m+m)V$

$\frac{1}{2}mV^2 = mgh$

$V^2 = \frac{m^2U^2}{(m+m)^2}$

$\frac{1}{2} \times \frac{m^2U^2}{(m+m)^2} = mgh$

$h = \frac{1}{2g}\left(\frac{mU}{m+m}\right)^2$

Candidate 2 evidence

QUESTION NUMBER	
4.(a)	$v(t) = 2\underline{i} + 6t\underline{j} - 10t\underline{k}$ $t = 3$ $= 2\underline{i} + 18\underline{j} - 30\underline{k}\ ms^{-1}$ $= \sqrt{2^2 + 18^2 + 30^2}$ $= 35\ ms^{-1}$
4.(b)	~~$\sqrt{2^2 + 2^2 + 18^2 + \ldots}$~~ $v^2 = 2^2 + 36t^2 + 100t^2$ $2500 = 2^2 + 136t^2$ $2498 = 136t^2$ $18.37 = t^2$ $t = -4.29$ $t = 4.29s$

QUESTION NUMBER

8.(a)

$r_b = 5\underline{i} + 2\underline{j}$ m $\qquad r_w = 60\underline{i} + 40\underline{j}$

$v_b = 4\underline{i} + \underline{j}$ ms^{-1}

$r_b(t) = r_b + v_b(t)$

to sub in

$= (5+4t)\underline{i} + (2+t)\underline{j}$

$_w r_b = r_b - r_w$

$= (4t-55)\underline{i} + (t-38)\underline{j}$

$r^2 = 16t^2 - 440t + 3025 + t^2 - 76t + 1444$

$= 17t^2 - 516t + 4469$

$\frac{d}{dt} r^2 = 34t - 516$

$\frac{d}{dt} = 0 = 34t - 516$

$516 = 34t$

$t = 15.2s$

$r^2 = 16(15.2)^2 - 516(15.2) + 4469$

$= 323$

$r = 18$ m

8.(b)

Whale dives

whale remains at rest

QUESTION NUMBER 10.(a)

C

$v^2 = u^2 + 2as$

$0 = 10.5^2 - 2gs_1$

$2gs_1 = 10.5^2$

$s_1 = \frac{10.5^2}{2g}$

$s_2 = s_1 - 2$

D

$v^2 = u^2 + 2as$

$0 = u^2 - 2gs_2$

$2gs_2 = u^2$

$2g(s_1 - 2) = u^2$

$2g\left(\frac{10.5^2}{2g} - 2\right) = u^2$

$10.5^2 - 4g = u^2$

$71.05 = u^2$

$u = 8.43 \text{ ms}^{-1}$

QUESTION NUMBER 10.(b)

C

$s = ut + \frac{1}{2}at^2$

$s_1 = 10.5t - 4.9t^2$

$s_1 - 2 = 8.43t - 4.9t^2$

$10.5t - 4.9t^2 - 2 = 8.43t - 4.9t^2$

$10.5t - 2 = 8.43t$

$2.07t = 2$

$t = 0.966$

$v_1 = u + at$

$= 10.5 - g \times 0.966$

$= 1 \text{ ms}^{-1}$

D

$v_2 = u + at$

$= 8.43 - g \times 0.966$

$= -1 \text{ ms}^{-1}$

QUESTION NUMBER 14.

$9\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 4y = 0$

$9k^2 + 12k + 4 = 0$

$(3k+2)(3k+2)$

$3k = -2$

$k = -\frac{2}{3}$

$y = (Ax+B)e^{px}$

$= (Ax+B)e^{-\frac{2}{3}x}$ $\quad y = (6-x)e^{-\frac{2}{3}x}$

$y = 6 \quad 6 = Be^0$

$x = 0 \quad 6 = B$

$\frac{dy}{dx} = Ae^{-\frac{2}{3}x} - \frac{2}{3}(Ax+B)e^{-\frac{2}{3}x}$

$\frac{dy}{dx} = -3 \quad -3 = Ae^0 - \frac{2}{3}Be^0$

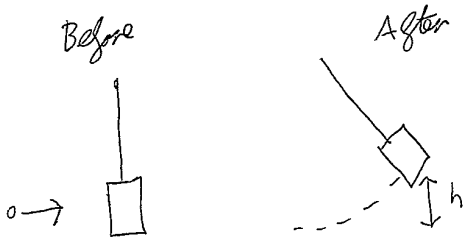
$x = 0 \quad -3 = A - \frac{2}{3} \times 3$

$-3 = A - 2$

$-1 = A$

QUESTION NUMBER

15.



$E_{k_1} = \frac{1}{2}mv^2$

$= \frac{1}{2}mv^2$

$E_k = 0$

$E_p = \frac{1}{2}mv^2$

$(M+m)gh = \frac{1}{2}mv^2$

$(M+m)h = \frac{1}{2g}mv^2$

$h = \frac{1}{2g}\frac{mv^2}{M+m}$

QUESTION NUMBER

17.(a)

$\int x \sin 2x \, dx$ $\quad u = x \quad v' = \sin 2x$

$\int uv' dx = uv - \int u'v \, dx + C \quad u' = 1 \quad v = -\frac{1}{2}\cos 2x$

$= \frac{x}{2}\cos 2x + \frac{1}{2}\int \cos 2x \, dx$

$= \frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x + C$

$= \frac{1}{2}(x\cos 2x + \frac{1}{2}\sin 2x) + C$

17.(b)

$y = \sqrt{x \sin 2x}$

$y^2 = x \sin 2x$

$V = \pi \int_a^b y^2 \, dx$

$= \pi \int_0^1 x \sin 2x$

$= \pi \left[\frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x \right]_0^1$

$= \pi(-0.21 + 0.23 - 0 - 0)$

$= 0.02\pi$ units^3

QUESTION NUMBER

18.(a)

$v = 2\sqrt{3gr}$

$E_{k_1} = \frac{1}{2}mv^2$

$= \frac{1}{2}m \times 12gr$

$= 6mgr$

$E_{p_1} = mgh$

$= mg \times r \times 2$

$= 2mgr$

$E_t = 8mgr$

$E_p = 0$

$E_t = E_{k_2}$

$8mgr$

$\frac{1}{2}mv^2 = 8mgr$

$v^2 = 16gr$

$v = 4\sqrt{gr}\ ms^{-1}$

18.(b) (i)

$T = 0$

F_c

mg

$E_{k_2} = \frac{1}{2}mgr\sin\theta$

$E_t = 2mgr$

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times m \times 4gr$

$E_t = 2mgr$

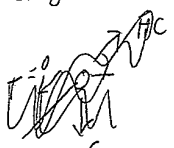
$\frac{mgr}{\sin\theta}$

$F_c \sin\theta = mg$

$\frac{mv^2}{r}\sin\theta = mg$

$v^2 \sin\theta = gr$

$\frac{v^2}{r}\sin\theta = g$

QUESTION NUMBER	
18.(b) (i) (cont)	 $\theta = 90^\circ$ $h = r$ $E_p = mgh$ $= mgr$ $E_k = \frac{1}{2}mv^2$ $E_{k_1} = mgr$ $v = 2\sqrt{gr}\ ms^{-1}$ $E_{k_1} = \frac{1}{2}mv^2$ $= \frac{1}{2}m \times 4gr$ $= 2mgr$  $E_k = mg(r-x)$ $\frac{1}{2}mv^2 = mg(r-x)$ $v^2 = 2g(r-x)$ $v = \sqrt{2g(r-x)}$ $h = r + x$ $E_p = mgr + mgx$  $mg\cos\theta = T$ m
18.(b) (ii)	ball towards centre

Candidate 3 evidence

QUESTION NUMBER

1.

$$\frac{48}{1000}\begin{pmatrix}16\\0\end{pmatrix}+\frac{32}{1000}\begin{pmatrix}0\\0\end{pmatrix}=\frac{48}{1000}\begin{pmatrix}4\\-8\end{pmatrix}+\frac{32}{1000}v_2$$

$$\begin{pmatrix}\frac{96}{125}\\0\end{pmatrix}=\begin{pmatrix}\frac{24}{125}\\-\frac{48}{125}\end{pmatrix}+\frac{32}{1000}v_2$$

$$v_2=\begin{pmatrix}\frac{96-24}{125}\\-\frac{48}{125}\end{pmatrix}$$

$$|v_2|=\sqrt{\left(\frac{72}{125}\right)^2+\left(-\frac{48}{125}\right)^2}$$

$$|v_2|=0.692\ ms^{-1}$$

$$v_2=\begin{pmatrix}\frac{72}{125}\\-\frac{48}{125}\end{pmatrix}$$

QUESTION NUMBER

3.(a)

$a = -g \quad v = U\sin\theta \quad s = 0$

$v = \int a\,dt = -gt + c$

$= -gt + U\sin\theta$

$s = \int v\,dt = -\frac{g}{2}t^2 + Ut\sin\theta + c$

$s = -\frac{g}{2}t^2 + Ut\sin\theta$

max s when $\frac{ds}{dt} = 0$

$0 = -gt + U\sin\theta$

$U\sin\theta = gt$

$t = \frac{U}{g}\sin\theta$

$\max s = -\frac{g}{2}\left(\frac{U^2\sin^2\theta}{g^2}\right) + U\sin\theta\left(\frac{U}{g}\sin\theta\right)$

$\max s = -\frac{U^2\sin^2\theta}{2g} + \frac{2U^2\sin^2\theta}{2g}$

$\max s = \frac{U^2\sin^2\theta}{2g}$

3.(b)

$H = \frac{40^2\sin^2(27)}{2\times 9.8} \qquad H = 50$

$H + h = \frac{1600\sin^2(27)}{19.6}$

$h = \frac{1600\sin^2(27)}{19.6} - 50$

$h = 16.825\ldots - 50$

$h = -33.17\text{m}$

QUESTION NUMBER

7.

$$f(t) = \frac{5t}{t^2+3}$$

$$f'(t) = \frac{5(t^2+3) - 5t(2t)}{(t^2+3)^2}$$

$$= \frac{5t^2+15-10t^2}{(t^2+3)^2}$$

$$= \frac{15-5t^2}{(t^2+3)^2}$$

$$f(t) = 0$$

$$0 = \frac{15-5t^2}{(t^2+3)^2}$$

$$t^4 + 6t^2 + 9 = 15 - 5t^2$$

$$t^4 + 11t^2 - 6 = 0$$

[redacted]

$$t^2 = \frac{-11 \pm \sqrt{11^2 - 4\times(-6)}}{2}$$

$t^2 = 0.52079\ldots$ or ~~$t^2 = -11.52\ldots$~~

$t = 0.7225$

QUESTION NUMBER

10.(a)

$a_C = -g$ $\quad a_D = -g$

$v_C = -gt + 10.5$ $\quad v_D = -gt + u$

$s_C = -\frac{1}{2}gt^2 + 10.5t$ $\quad s_D = -\frac{1}{2}gt^2 + ut + 2$

max height when $v = 0$

$0 = -gt + 10.5$ $\quad 0 = -gt + u$

$t = \frac{10.5}{g}$ $\quad -gt = u$

$t = -\frac{u}{g}$

$s_C = -\frac{1}{2}g\left(\frac{10.5}{g}\right)^2 + 10.5\left(\frac{10.5}{g}\right)$ $\quad s_D = -\frac{1}{2}g\left(\frac{u}{g}\right)^2 + u\left(\frac{u}{g}\right) + 2$

$= -\frac{1}{2}\frac{10.5^2}{g} + \frac{10.5^2}{g}$ $\quad s_D = -\frac{1}{2}\frac{u^2}{g} + \frac{u^2}{g} + 2$

$= \frac{1}{2}\frac{10.5^2}{g}$ $\quad s_D = \frac{1}{2}\frac{u^2}{g} + 2$

$\frac{1}{2}\frac{10.5^2}{g} = \frac{1}{2}\frac{u^2}{g} + 2$

$\frac{10.5^2}{g} = \frac{u^2}{g} + 4$

$10.5^2 = u^2 + 4g$

$u = \sqrt{10.5^2 - 4g}$

$u = 8.43\ ms^{-1}$

$= \frac{7\sqrt{145}}{10}$

QUESTION NUMBER

10.(b)

$s_C = s_D$

$$-\frac{1}{2}gt^2 + 10.5t = -\frac{1}{2}gt^2 + \frac{7\sqrt{145}}{10}t + 2$$

$$gt^2 - 21t = gt^2 - \frac{14\sqrt{145}}{10}t - 4$$

$$21t = \frac{14\sqrt{145}}{10}t + 4$$

$$t = \frac{4}{21 - \frac{14\sqrt{145}}{10}}$$

$$t = 0.966\ldots$$

$$t = \frac{15 + \sqrt{145}}{28}$$

$$v_C = -g\left(\frac{15+\sqrt{145}}{28}\right) + 10.5 \qquad v_D = -g\left(\frac{15+\sqrt{145}}{28}\right) + \frac{7\sqrt{145}}{10}$$

$$= -52.718\ldots \qquad = -1.035$$

?

QUESTION NUMBER

14.

$\frac{d^2y}{dx^2} + \frac{4}{3}\frac{dy}{dx} +$

$9\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 4 = 0$

$(k - \frac{2}{3})(k - \frac{2}{3}) = 0$

$k = \frac{2}{3}$

$y = (Ax + B)e^{+\frac{2}{3}x}$

$\frac{dy}{dx} = Ae^{\frac{2}{3}x} + \frac{2}{3}(Ax + B)e^{\frac{2}{3}x}$

$6 = (A(0) + B)e^{0}$

$6 = B$

$-3 = Ae^{0} + \frac{2}{3} \times 6 \times e^{0}$

$-3 = A + 4$

$A = -7$

$y = (6 - 7x)e^{\frac{2}{3}x}$

$-7e^{\frac{2}{3}x} + \frac{2}{3}(6 - 7x)e^{\frac{2}{3}x}$

$-7 \times \frac{2}{3}e^{\frac{2}{3}x} + \frac{2}{3}(-7)e^{\frac{2}{3}x} + \frac{4}{9}(6 - 7x)$

QUESTION NUMBER

15. E_k bullet = E_p block + bullet at max height

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2}mu^2$

$\frac{1}{2}mu^2 = (M+m)gh$

$h = \frac{1}{2}\frac{mu^2}{M+m} \times \frac{1}{g}$

$h = \frac{1}{2g}\left(\frac{mu^2}{M+m}\right)$

QUESTION NUMBER

18.(a) for $\frac{mv^2}{r} = mg$ $R_{in} > 0$

$0 = mg + \frac{mv^2}{r}$

$\frac{m \times 12gr}{r} = mg$

$mg = -\frac{mv^2}{r}$

$mg = -\frac{m \times 12gr}{r}$

$mg = -12mg$

$1 = -12$ wow!

$\frac{mv^2}{r} = mg$

$mv^2 = -12mg$

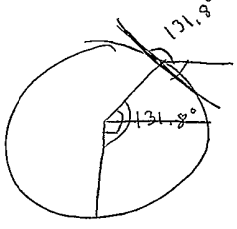
$v^2 = -12rg = 12rg$ in opp dir

$v = \sqrt{12rg}$

$= 2\sqrt{3rg}$ nice!

$2\sqrt{3gr}$

18.(b) (ii) $\theta = 131.8°$



the particle will follow a parabola launched initially at an angle of 131.8° to the positive horizontal.