Candidate 4 evidence

$$\begin{pmatrix}0.768\\0\end{pmatrix} - \begin{pmatrix}0.192\\-0.384\end{pmatrix}$$

QUESTION NUMBER

1.

$m_1 = 0.048\text{kg}$

$m_2 = 0.032\text{kg}$

$$\boxed{0.048} + \boxed{0.032} \rightarrow \boxed{0.048} + \boxed{0.032}$$

$$\begin{pmatrix}16\\0\end{pmatrix}\text{ms}^{-1} \qquad \begin{pmatrix}0\\0\end{pmatrix} \qquad \begin{pmatrix}4\\-8\end{pmatrix}\text{ms}^{-1} \qquad V_2 = ?$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2V_2$$

$$0.048\begin{pmatrix}16\\0\end{pmatrix} + 0.032\begin{pmatrix}0\\0\end{pmatrix} = 0.048\begin{pmatrix}4\\-8\end{pmatrix} + 0.032V_2$$

$$\begin{pmatrix}0.768\\0\end{pmatrix} + \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}0.192\\-0.384\end{pmatrix} + 0.032V_2$$

$$\begin{pmatrix}0.576\\0.384\end{pmatrix} = 0.032V_2$$

$$V_2 = \begin{pmatrix}18\\12\end{pmatrix} \qquad |\underline{V}_2| = \sqrt{18^2 + 12^2}$$

$$= \underline{21.6\text{ms}^{-1}}$$

2.

$f(x) = \ln(\sec 2x)$

$$f'(x) = \frac{2\cancel{\sec 2x}\tan 2x}{\cancel{\sec 2x}}$$

$$= \underline{2\tan 2x}$$

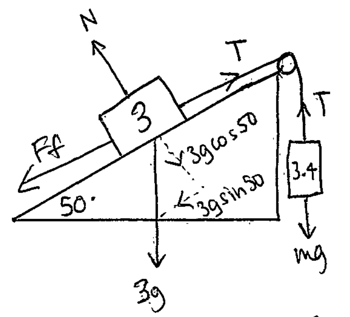
$f(x) = \sec(2x)$

$f'(x) = \sec 2x \tan 2x \cdot 2$

$= 2\sec 2x \tan 2x$

QUESTION NUMBER

16.(a)



$Ff = \mu N$

$T = 3g\sin 50 + Ff$ ~~T = mg~~ ~~= 3.4g~~ $T = mg$
$= 3.4g$

$3.4g = 3g\sin 50 + \mu N$

$3.4g = 3g\sin 50 + \mu(3g\cos 50)$

$\mu(3g\cos 50) = 41.03387\ldots$

$\mu = \underline{\underline{1.45}}$

~~3.4g − 3gsin50 / 3gcos50~~

16.(b)

$F_{un} =$ ~~[illegible]~~ $F - Ff$

$F_{un} = -(\mu N)$

$ma = -(1.45 \times (3g\cos 50))$

$3a = -41.14$

$a = -13.7$

$a = -13.7$

$u = 0$

$s = 8m$

$t = ?$

~~v = u + at~~ ~~v = −13?~~

$v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times (-13.7) \times 8$

$v = -14.81 ms^{-1}$

$v = u + at$

$-14.81 = -13.7 \times t$

$t = \underline{\underline{1.1s}}$

QUESTION NUMBER

17.(a)

$\int x\sin 2x \, dx$ $\quad f = x \quad g = -\frac{1}{2}\cos 2x$

$f' = 1 \quad g' = \sin 2x$

$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$

$= x\cdot(-\frac{1}{2}\cos 2x) - \int -\frac{1}{2}\cos 2x \, dx$

$= -\frac{1}{2}x\cos 2x - (-\frac{1}{4}\sin 2x) + c$

$= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + c$

$= \frac{1}{2}(-x\cos 2x + \frac{1}{2}\sin 2x + 2c)$

17.(b)

$V = \pi \int_{x_1}^{x_2} y^2 \, dx$ $\quad y^2 = x\sin 2x$

$V = \pi \int_0^1 x\sin 2x \, dx$

$V = \pi \left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^1$

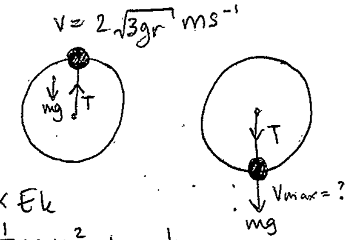
$V = \pi\left((-\frac{1}{2}\cos 2 + \frac{1}{4}\sin 2) - (0 + \frac{1}{4}\sin 0)\right)$

$= \pi(0.435)$

$V = 0.435\pi$ units^3

QUESTION NUMBER

18.(a)



max Ek

$= \frac{1}{2}mu^2 + mgh$

$= \frac{1}{2} \times m \times (2\sqrt{3gr})^2 + mg(2r)$

$= \frac{1}{2}m(4(3gr)) + 2rmg$

$= \frac{1}{2}m12gr + 2rmg$

$= 6rmg + 2rmg$

$= 8rmg$

$Ek = \frac{1}{2}mv^2$

$8r\cancel{m}g = \frac{1}{2}\cancel{m}v^2$

$8rg = \frac{1}{2}v^2$

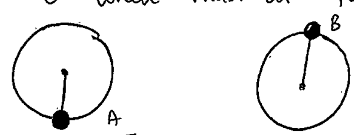
$v^2 = 16rg$

$V_{max} = \sqrt{16rg}$

$= 4\sqrt{rg}$ ms^{-1}

18.(b) (i)

T = 0 when mass at top of circle.



$V = 2\sqrt{gr}$ ms^{-1}

At A

$Ek = \frac{1}{2}mv^2$ $Ep = 0$

At B

$Ek = \frac{1}{2}mv^2$ $Ep = mgh$

$\frac{1}{2}\cancel{m}u^2 = \frac{1}{2}\cancel{m}v^2 + \cancel{m}gh$

$\frac{1}{2}u^2 = \frac{1}{2}v^2 + gh$

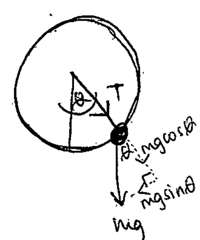
$u^2 = v^2 + 2gh$

$(2\sqrt{gr})^2 = v^2 + 2gh$

$4gr = v^2 + 2gh$

$v^2 = 4gr - 2gh = 2g(2r - h)$

$\frac{r}{m}(T + mg\cos\theta) = 4gr - 2gh$



$T + mg\cos\theta = \frac{mv^2}{r}$

$r(T + mg\cos\theta) = mv^2$

$\frac{r}{m}(T + mg\cos\theta) = v^2$

QUESTION NUMBER

18.(b)(i)(cont)

$$2g(2r-h) = \frac{r}{m}(T + mg\cos\theta)$$

$$\frac{2mg(2r-h)}{r} = T + mg\cos\theta$$

$$\frac{2mg(2r-h)}{r} - T = mg\cos\theta$$

$$\cos\theta = \frac{\frac{2mg(2r-h)}{r} - T}{mg}$$

18.(b)(ii)

It falls back down

Candidate 5 evidence

QUESTION NUMBER

1.

$m_1 = 0.048$

$u_1 = \begin{pmatrix} 16 \\ 0 \end{pmatrix} cms^{-1}$

$m_2 = 0.032$

$v_1 = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$

$u_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$v_2 = ?$

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\cancel{0.048(16) + \overset{0.032}{m_2}(0) = \overset{0.04}{m_1}(4\sqrt{5}) + m_2 v_2}$

$0.048(16) + 0.032(0) = 0.048(4\sqrt{5}) + 0.032(v_2)$

$v_2 = \dfrac{0.048(16) - 0.048(4\sqrt{5})}{0.032}$

$\therefore v_2 = 10.5835\ldots$

$\underline{v_2 = 10.6 cms^{-1}}$

QUESTION NUMBER

13.

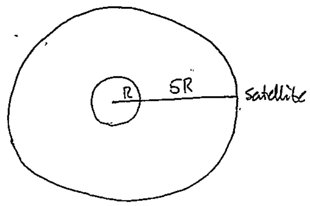
$T = 12\pi\sqrt{2R}$

$r_p = R$

$h_s = 5R$

$r_0 = 6R$

$g_s = 3$



$F = \frac{GMm}{r^2}$

$\cancel{m}g = \frac{GM\cancel{m}}{r^2}$

$\cancel{m}a = \cancel{m}\omega^2 r$

$\cancel{m}g = \frac{GM\cancel{m}}{R^2}$

$g = \frac{GM}{36R^2}$

$\omega = \frac{1}{T}$

$3 = \frac{GM}{R^2}$

$3 = kg$

$3 = kg$

$\frac{GM}{R^2} = k\frac{GM}{36R^2}$

$3 = 36g$

$R^2 = \frac{1}{k}36R^2$

$\underline{g = \frac{1}{12}}$

$k = 36$

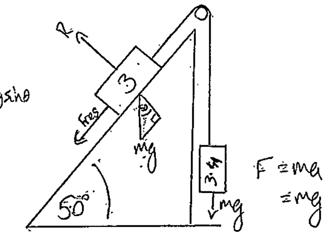
$F = ma$

QUESTION NUMBER	
16.(a)	see below
16.(b)	see below

16.(a)

a) $\sum_{\text{horizontal}} = F - F_{res} - mg\sin\theta$

$\sum_{\text{vert}} = R - mg\cos\theta$



$F_{res} = \mu R$

$R = mg\cos\theta$

$F_r = \mu mg\cos\theta$

$m_1 = 3$

$m_2 = 3.4$

$\sum h = mg - \mu mg\cos\theta - mg\sin\theta = 0$

$0 = 3.4g - \mu 3g\cos 50 - 3g\sin 50$

$\mu = \dfrac{3.4g - 3g\sin 50}{3g\cos 50}$

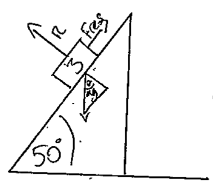
$\mu = 0.5714...$

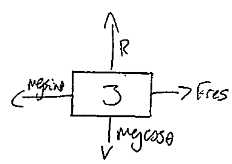
$\mu = 0.57$

16.(b)

$\sum v = R - mg\cos\theta$

$R = mg\cos\theta$





$\sum h = mg\sin\theta - F_{res}$

$= ma$

$\cancel{m}a = \cancel{m}g\sin(50) - 0.57\cancel{m}g\cos(50)$

$a = g\sin(50) - 0.57g\cos(50)$

$a = 9.8\sin 50 - 0.57(9.8)\cos 50$

$a = 3.91662...$

$F_r = \mu R$

$= 0.57(mg\cos\theta)$

$= 0.57(3(9.8)\cos 50)$

$= 10.771...$

$= A$

$s = ut + \frac{1}{2}at^2$

$8 = 0t + \frac{1}{2}(3.916...)t^2$

$t^2 = \dfrac{8}{\frac{1}{2}(3.916...)}$

$t = \sqrt{\text{Answer}}$

$t = 2.02117...$

$t = 2.02\,s$

QUESTION NUMBER

17.(a)

$\int x \sin 2x \, dx$ $\qquad u = x \quad v = \sin 2x$

$\qquad u' = 1 \quad \int v = -\frac{1}{2}\cos 2x$

$= u\int v - \int u' \int v$

$= x\left(-\frac{1}{2}\cos 2x\right) - \int 1\left(-\frac{1}{2}\cos 2x\right) dx$

$= -\frac{x}{2}\cos 2x + \frac{1}{2}\int \cos 2x \, dx$

$= -\frac{x}{2}\cos 2x + \frac{1}{2}\left(\frac{1}{2}\sin 2x\right)$

$= \frac{1}{4}\sin 2x - \frac{x}{2}\cos 2x$

$= \frac{1}{4}\left(\sin 2x - 2x\cos 2x\right)$

17.(b)

$y = \sqrt{x \sin 2x}$ $\qquad$ Limits $x = 0, x = 1$

$V = \pi \int y^2 dx$

~~$V = \pi \int_0^1 (x \sin 2x)^{\frac{1}{2}} dx$~~

~~$= \pi \cdot \frac{2}{3}(x \sin 2x$~~

$V = \pi \int_0^1 x \sin 2x \, dx$

$= \pi\left[\frac{1}{4}\sin 2x - \frac{x}{2}\cos 2x\right]_0^1$

$= \pi\left[\left(\frac{1}{4}\sin 2 - \frac{1}{2}\cos 2\right) - \left(\frac{1}{4}\sin 0 - \frac{0}{2}\cos 0\right)\right]$

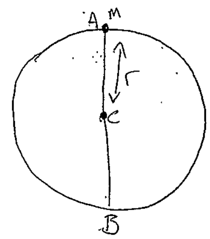
~~$= \pi[$~~

$= \frac{\pi}{4}\sin 2 - \frac{\pi}{2}\cos 2$

$= 1.36784...$

$= 1.37 \text{ units}^3$

QUESTION NUMBER	
18.(a)	



$v_A = 2\sqrt{3gr}$

max v at B

$E_A = \frac{1}{2}mv^2 + mgh$

$= \frac{1}{2}m(12gr) + mg(2r)$

$E_B = \frac{1}{2}mv^2 + mgh$

$= \frac{1}{2}mv_B^2 + mg(0)$

$E_A = E_B$

$\Rightarrow \frac{1}{2}mv_B^2 = \frac{1}{2}m(12g^2r^2) + \frac{4}{2}mgr$

$v_B^2 = 12(gr)^2 + 4gr$

$\cancel{v_B^2 = gr(12gr+4)}$

$\cancel{gr = \frac{v_B^2}{12gr+4}}$

$v_B^2 = 12gr + 4gr \Rightarrow v_B^2 = 16gr$

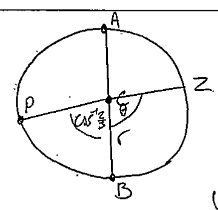
$\cancel{gr = v_B^2}$

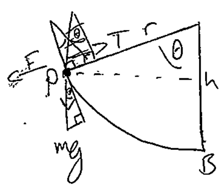
$\underline{v_B = 4\sqrt{gr}}$

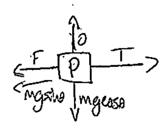
18.(b) (i)

$V_B = 2\sqrt{gr}$

find θ when $T=0$







$\omega^2 r = \frac{v^2}{r}$

$\omega^2 = \frac{v^2}{r^2}$

$\omega = \frac{v}{r}$

$E_B = \frac{1}{2}mv^2 + mgh$

$= \frac{1}{2}m(2\sqrt{gr})^2 + 0$

$= \frac{1}{2}m \cdot 4gr$

$= 2mgr$

$E_P = E_B$

$\therefore E_P = \frac{1}{2}mv^2 + mgh$

$v=0 \quad 2mgr = 0 + mgh$

$\therefore h = 2r$

$h = r(1-\cos\theta) \quad r\cos\theta = -r$

$2r = r - r\cos\theta \quad \cos\theta = -1$

~~Σvert =~~

$T=0$ when $F=0$

$F = m\omega^2 r$

find when $\omega = 0$

$F=0$ when $\omega=0$

QUESTION NUMBER	
18.(b) (i) (cont)	$\sum_{\text{perp}} = T - F - mg\sin\theta$ $T=0$ $\quad T = F + mg\sin\theta$ $\quad 0 = F + mg\sin\theta$ $F = m\omega^2 r$ $\quad \cancel{m}\omega^2 r = \cancel{m}g\sin\theta$ $\omega^2 r = g\sin\theta$ $\sin\theta = \frac{\omega^2 r}{g}$ $\omega^2 r = \frac{v^2}{r}$ $\quad \sin\theta = \frac{v^2}{rg}$
18.(b) (ii)	The particle will travel back towards $\underline{B}$ (the lowest part of the circle) ?. When the arc is small, the motion of the particle could be described as simple harmonic

Candidate 6 evidence

QUESTION NUMBER

1.

$m_1 = 48$ grams
$\underline{u}_1 = 16\underline{i}$ cms^{-1}
$m_2 = 32$ grams
$\underline{v}_1 = (4\underline{i} - 8\underline{j})cms^{-1}$

By conservation of momentum,

$$m_1\underline{u}_1 + m_2\underline{u}_2 = m_1\underline{v}_1 + m_2\underline{v}_2$$

$$48(16\underline{i}) + 32(0\underline{i} + 0\underline{j}) = 48(4\underline{i} - 8\underline{j}) + 32\underline{v}$$

$$768\underline{i} = (192\underline{i} - 384\underline{j}) + 32\underline{v}$$

$$32\underline{v} = 576\underline{i} + 384\underline{j}$$

$$\underline{v} = (18\underline{i} + 12\underline{j})\ cms^{-1}$$

$$|\underline{v}| = \sqrt{18^2 + 12^2} = \sqrt{468} = 6\sqrt{13} = 21.633\ldots$$

$$= \underline{\underline{21.6}}\ cms^{-1}\ (3sf)$$

2.

$$f(x) = \ln(\sec 2x)$$

$$f'(x) = \frac{1}{\sec 2x} \cdot \frac{d}{dx}(\sec 2x)$$

$$f'(x) = \frac{1}{\sec 2x} \cdot 2\sec 2x \tan 2x$$

$$f'(x) = \frac{2\sec 2x \tan 2x}{\sec 2x}$$

$$f'(x) = \underline{\underline{2\tan 2x}}$$

QUESTION NUMBER

3.(a)

$u_y = u\sin\theta$

$a = -g$

max height reached when $v_y = 0\,ms^{-1}$

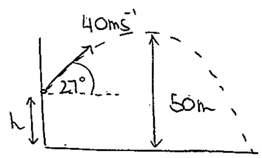
$\uparrow \quad v^2 = u^2 + 2as$

$0^2 = (U\sin\theta)^2 + 2(-g)H$

$2gH = U^2\sin^2\theta$

$$H = \frac{U^2\sin^2\theta}{2g}$$ (as required)

3.(b)



using part (a)

$$H = \frac{40^2\sin^2 27^\circ}{2(9.8)}$$

$H = 16.825\ldots m$

$h = 50 - 16.825\ldots$

$h = 33.174\ldots$

$h = \underline{\underline{33.2\,m}}$ (3sf)

QUESTION NUMBER	
4.(a)	$\underline{s} = 2t\,\underline{i} + 3t^2\underline{j} - 5t^2\underline{k}$ $\underline{v} = \frac{d\underline{s}}{dt}$ $\underline{v} = 2\underline{i} + 6t\underline{j} - 10t\,\underline{k}$ when $t = 3$ $\underline{v} = 2\underline{i} + 6(3)\underline{j} - 10(3)\underline{k}$ $\underline{v} = (2\underline{i} + 18\underline{j} - 30\underline{k})\ ms^{-1}$
4.(b)	$\text{speed} = \lvert\underline{v}\rvert$ Find t when speed $= 50ms^{-1}$ $50 = \sqrt{(2)^2 + (6t)^2 + (-10t)^2}$ $50 = \sqrt{4 + 36t^2 + 100t^2}$ $50 = \sqrt{4 + 136t^2}$ $2500 = 4 + 136t^2$ $136t^2 = 2496$ $t^2 = \frac{312}{17}$ $t = \pm 4.284\ldots$ $t \geq 0 \therefore t = \underline{4.28}\ s\ (3sf)$

QUESTION NUMBER

5.

$\int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x \, dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$= \int_0^{\frac{\pi}{3}} (\tan x)^2 \sec^2 x \, dx$

$= \int_0^{\sqrt{3}} u^2 \, du$

when $x = 0$

$u = \tan 0 = 0$

when $x = \frac{\pi}{3}$

$u = \tan\frac{\pi}{3} = \sqrt{3}$

$= \left[\frac{u^3}{3}\right]_0^{\sqrt{3}}$

$= \left(\frac{(\sqrt{3})^3}{3}\right) - \frac{0^3}{3}$

$= \frac{3\sqrt{3}}{3} - 0$

$= \sqrt{3}$

QUESTION NUMBER

6.(a)

$T = \frac{\pi}{8}$ s

speed at equillibrium position is $2ms^{-1}$

$\Rightarrow$ $|v|_{max} = 2ms^{-1}$

$T = \frac{2\pi}{\omega}$

$\frac{\pi}{8} = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{\frac{\pi}{8}}$

$\omega = 16 \text{ rad}s^{-1}$

$|v|_{max} = \omega a$

$2 = 16a$

$a = 0.125m$

6.(b)

$|\ddot{x}|_{max} = \omega^2 a$

$= 16^2(0.125)$

$= 32ms^{-2}$

QUESTION NUMBER

7.

$f(t) = \frac{5t}{t^2+3}$ Quotient Rule

$u = 5t$ $v = t^2+3$

$u' = 5$ $v' = 2t$

$f'(t) = \frac{vu' - uv'}{v^2}$

$f'(t) = \frac{5(t^2+3) - 2t(5t)}{(t^2+3)^2}$

$f'(t) = \frac{5t^2+15-10t^2}{(t^2+3)^2}$

$f'(t) = \frac{15-5t^2}{(t^2+3)^2}$

$f'(t) = 0$

$\frac{15-5t^2}{(t^2+3)^2} = 0$

$15 - 5t^2 = 0$

$5t^2 = 15$

$t^2 = 3$

$t = \pm\sqrt{3}$

$t > 0 \therefore t = \sqrt{3}$

QUESTION NUMBER

8.(a)

Let B = boat
W = whale

Boat

$\underline{r}_0 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ m

$\underline{v}_B = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ ms^{-1}

Whale

$\underline{r}_W = \begin{pmatrix} 60 \\ 40 \end{pmatrix}$ m

$\underline{r}_B = \underline{v}_B t + \underline{r}_0$

$\underline{r}_B = \begin{pmatrix} 4 \\ 1 \end{pmatrix} t + \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

$\underline{r}_B = \begin{pmatrix} 4t \\ t \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

$\underline{r}_B = \begin{pmatrix} 4t+5 \\ t+2 \end{pmatrix}$

${}_B\underline{r}_W = \underline{r}_B - \underline{r}_W$

$= \begin{pmatrix} 4t+5 \\ t+2 \end{pmatrix} - \begin{pmatrix} 60 \\ 40 \end{pmatrix}$

$= \begin{pmatrix} 4t-55 \\ t-38 \end{pmatrix}$

$d = |{}_B\underline{r}_W| = \sqrt{(4t-55)^2 + (t-38)^2}$.

$d^2 = |{}_B\underline{r}_W|^2 = (4t-55)^2 + (t-38)^2$

$\frac{d}{dt}d^2 = 2(4)(4t-55) + 2(t-38)$

$= 32t - 440 + 2t - 76$

$= 34t - 516$

For closet distance, $\frac{d}{dt}d^2 = 0$

$34t - 516 = 0$

$t = 15.176\ldots s$

$d^2 = (4(15.176\ldots) - 55)^2 + (15.176\ldots - 38)^2$

$d^2 = 553.470\ldots$

$d = 23.525\ldots$

$d = \underline{\underline{23.5m}}$ (3sf)

8.(b)

~~Whale does not move position i.e. water tide does not cause position of whale to alter~~

Air resistance is negilible, i.e. wind resistance.

QUESTION NUMBER	
9.(a)	$f'(t) = \dfrac{4t+17}{2t^2+17t+8}$ $\quad f(0) = \ln 4$ $\quad$ of the form $\int \dfrac{f'(x)}{f(x)}\,dx$ $f(t) = \int f'(t)\,dt$ $f(t) = \int \dfrac{4t+17}{2t^2+17t+8}\,dt$ $\quad \dfrac{d}{dt}(2t^2+17t+8) = 4t+17$ $f(t) = \ln\lvert 2t^2+17t+8\rvert + c$ $\ln\lvert 2(0)^2+17(0)+8\rvert + c = \ln 4$ $\ln 8 + c = \ln 4$ $c = \ln 4 - \ln 8$ $c = \ln \frac{1}{2}$ $c = -\ln 2$ $f(t) = \ln\lvert 2t^2+17t+8\rvert - \ln 2$ $f(t) = \ln\left(\dfrac{2t^2+17t+8}{2}\right)$ ~~$f(t) = \ln\lvert\frac{2t^2+17t+8}{2}\rvert$~~
9.(b)	If $v = f'(t)$ $\quad s = \int v\,dt$ $s = f(t)$ $\quad s = \int f'(t)\,dt$ $s = f(t)$ $s = \ln\left(\dfrac{2t^2+17t+8}{2}\right)$ when $t = 3$ $s = \ln\left(\dfrac{2(3)^2+17(3)+8}{2}\right)$ $s = \ln \frac{77}{2}$ $s = 3.650\ldots$ $s = 3.65\text{m}$ (3sf)

QUESTION NUMBER

10.(a)

$g = 9.8\,ms^{-2}$ ↓ ↑ $u = 10.5\,ms^{-1}$

Particle C

max height when $v = 0\,ms^{-1}$

$v^2 = u^2 + 2as$

$0^2 = 10.5^2 + 2(-9.8)s$

$19.6s = 110.25$

$s = \frac{110.25}{19.6} = 5.625m$

$g = 9.8\,ms^{-2}$ ↓ ↑ u ↑ $2m$

Particle D

max height when $v = 0\,ms^{-1}$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2(-9.8)s$

$19.6s = u^2$

$s = \frac{u^2}{19.6}$

Particle D fired 2m above ground $\Rightarrow$ max height above ground $s = \frac{u^2}{19.6} + 2$

(displacement from horizontal ground)

Let u = initial speed of particle D

$\frac{u^2}{19.6} + 2 = 5.625$

$u^2 = 71.05$

$u = 8.429\ldots$

$u = \underline{\underline{8.43\,ms^{-1}}}$ (3sf)

Take ↑ as +ve direction

QUESTION NUMBER

10.(b)

Particle C

$s = ut + \frac{1}{2}at^2$

$s = 10.5t + \frac{1}{2}(-9.8)t^2$

$s = 10.5t - 4.9t^2$

Particle D

$s = ut + \frac{1}{2}at^2$

$s = \sqrt{71.05}\ t + \frac{1}{2}(-9.8)t^2$

$s = \sqrt{71.05}t - 4.9t^2$

displacement from grand

$s = \sqrt{71.05}t - 4.9t^2 + 2$

C and D at same height when

$10.5t - 4.9t^2 = \sqrt{71.05}\,t - 4.9t^2 + 2$

$10.5t = \sqrt{71.05}\ t + 2$

$t(10.5 - \sqrt{71.05}) = 2$

$t = 0.9657 \ldots s$

Particle C

$v = u + at$

$v = 10.5 - 9.8t$

when $t = 0.965\ldots s$

$v = 10.5 - 9.8(0.965..)$

$v = 1.035\ldots$

$v = 1.04\,ms^{-1}$ upwards (3sf)

Particle D

$v = u + at$

$v = \sqrt{71.05} - 9.8t$

when $t = 0.965\ldots s$

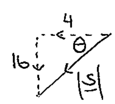
$v = \sqrt{71.05} - 9.8(0.965..)$

$v = -1.035\ldots$

$v = -1.04\,ms^{-1}$ (3sf)

$\Rightarrow$ $1.04\,ms^{-1}$ downwards

$\therefore$ when $t = 0.9657..\ s$ ie particles c and D are at same height speed of both particles is $1.04\,ms^{-1}$ (3sf) but particle C is travelling upwards and particle D is travelling downwards i.e same speed but are moving in opposite directions

QUESTION NUMBER	
11.(a)	$\underline{F} = 0.3\underline{i} + 0.5\underline{j}$ $(16,4) = 16\underline{i} + 4\underline{j}$ $\quad (12,20) = 12\underline{i} + 20\underline{j}$ $\underline{s} = (12\underline{i} + 20\underline{j}) - (16\underline{i} + 4\underline{j})$ $\underline{s} = -4\underline{i} + 16\underline{j}$ $WD = \underline{F} \cdot \underline{s}$ $= (0.3\underline{i} + 0.5\underline{j}) \cdot (-4\underline{i} + 16\underline{j})$ $= 0.3(-4) + 0.5(16)$ $= \underline{\underline{6.8J}}$
11.(b)	 $\tan\theta = \frac{O}{A}$ $\tan\theta = \frac{16}{4}$ $\tan\theta = 4$ $\theta = \tan^{-1}(4)$ $\theta = 75.963\ldots^\circ$ $\lvert\underline{s}\rvert = \sqrt{4^2+16^2}$ $= 16.492\ldots m$ $WD = F\cos\theta\, s$ $6.8 = F(\cos 75.963..^\circ)(16.492..)$ $F = \frac{6.8}{4}$ $F = \underline{\underline{1.7N}}$

QUESTION NUMBER

12.

$x^3 + y^2 + 2x - 4y = 33$

$\frac{d}{dx}x^3 + \frac{d}{dx}y^2 + \frac{d}{dx}2x - \frac{d}{dx}4y = \frac{d}{dx}33$

$3x^2 + 2y\frac{dy}{dx} + 2 - 4\frac{dy}{dx} = 0$

$(4-2y)\frac{dy}{dx} = 3x^2 + 2$

$\frac{dy}{dx} = \frac{3x^2+2}{4-2y}$

$(2, k)$

when $x = 2$

$2^3 + y^2 + 2(2) - 4y = 33$

$8 + y^2 + 4 - 4y = 33$

$y^2 - 4y - 21 = 0$

$(y+3)(y-7) = 0$

$y = -3, y = 7$

$\Rightarrow k = -3, k = 7$

when $k = -3$

$\frac{dy}{dx} = \frac{3(2)^2+2}{4-2(-3)}$

$= \frac{14}{10}$

$= \frac{7}{5} > 0$ i.e +ve gradient

when $k = 7$

$\frac{dy}{dx} = \frac{3(2)^2+2}{4-2(7)}$

$= \frac{-7}{5} < 0$

i.e −ve gradient

$\therefore$ For posive gradient $\underline{\underline{k = -3}}$

QUESTION NUMBER

13.

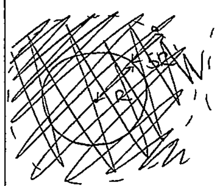
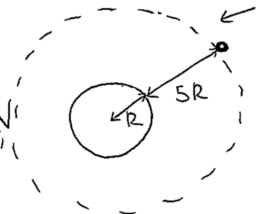


diagram not to scale



At satellite distance from centre of planet $= R + 5R$
$= 6R$

At surface of planet

$F = \frac{GMm}{r^2}$

$mg = \frac{GMm}{r^2}$

$m(3) = \frac{GMm}{R^2}$

$3R^2 = GM$

At satellite

$F = \frac{GMm}{r^2}$ $\quad F = mr\omega^2$

$mr\omega^2 = \frac{GMm}{r^2}$

$m(6R)\omega^2 = \frac{GMm}{(6R)^2}$

$(6R)^3\omega^2 = GM$

$(6R)^3\omega^2 = 3R^2$

$\omega^2 = \frac{3R^2}{216R^3}$

$\omega^2 = \frac{1}{72R}$

$\omega = \sqrt{\frac{1}{72R}}$

$\omega = \frac{1}{6\sqrt{2R}}$

$T = \frac{2\pi}{\omega}$

$T = \frac{2\pi}{\frac{1}{6\sqrt{2R}}}$

$T = 2\pi \times 6\sqrt{2R}$

$T = 12\pi\sqrt{2R}$ seconds

(as required)

QUESTION NUMBER

14.

$9\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 4y = 0$

AE : $9D^2 + 12D + 4 = 0$

$(3D+2)^2 = 0$

$D = -\frac{2}{3}$ (2 real and equal roots)

CF : $y = (A+Bx)e^{-\frac{2}{3}x}$

GS = CF

GS : $y = (A+Bx)e^{-\frac{2}{3}x}$

$u = A + Bx \quad v = e^{-\frac{2}{3}x}$

$u' = B \quad v' = -\frac{2}{3}e^{-\frac{2}{3}x}$

$\frac{dy}{dx} = -\frac{2}{3}e^{-\frac{2}{3}x}(A+Bx) + Be^{-\frac{2}{3}x}$

when $x = 0, y = 6$

$6 = (A + B(0))e^{-\frac{2}{3}(0)}$

$A = 6$

when $x = 0, \frac{dy}{dx} = -3$

$-3 = -\frac{2}{3}e^{-\frac{2}{3}(0)}(A+B(0)) + Be^{-\frac{2}{3}(0)}$

$-3 = -\frac{2}{3}A + B$

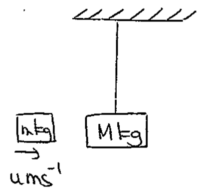
$-3 = -\frac{2}{3}(6) + B$

$B = 1$

PS : $\underline{\underline{y = (6+x)e^{-\frac{2}{3}x}}}$

QUESTION NUMBER

15.



Let V = speed of combined bullet and block

By conservation of momentum,

$$m_1u_1 + m_2u_2 = (m_1+m_2)v$$

$$mu + M(0) = (M+m)V$$

$$mu = (M+m)V$$

$$V = \frac{mu}{(M+m)}$$

Reaches height h m when speed of combined mass $= 0\text{ms}^{-1}$

By conservation of energy,

KE loss = PE gained

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2}(M+m)\left(\frac{mu}{(M+m)}\right)^2 = (M+m)gh$$

cancel out $(M+m)$

$$\frac{1}{2}\left(\frac{mu}{M+m}\right)^2 = gh$$

$$h = \frac{1}{2g}\left(\frac{mu}{M+m}\right)^2$$

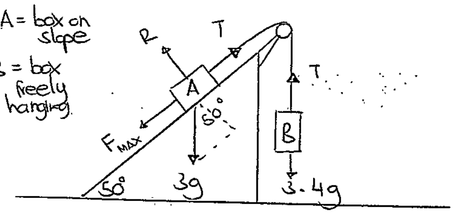
(as required)

QUESTION NUMBER

16.(a)

Let A = box on slope

B = box freely hanging



For B

resolve (↑) with N2L

$T - 3.4g = 3.4(0)$

$T = 3.4g$

$T = 33.32\,N$

For A

resolve perpendicular (↖)

$R = 3g \cos 50°$

$F = \mu R$

$F_{MAX} = \mu(3g\cos 50°)$

resolve parallel (↗) with N2L

$F_{un} = ma$

$T - 3g\sin 50° - F_{MAX} = 3(0)$

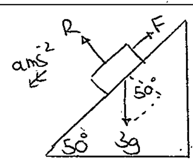
$33.32 - 3(9.8)\sin 50° - \mu(3(9.8)\cos 50°) = 0$

$$\mu = \frac{33.32 - 3(9.8)\sin 50°}{3(9.8)\cos 50°}$$

$\mu = 0.5714\ldots$

$\mu = \underline{\underline{0.571}}$ (3sf)

16.(b)



resolve (↖)

$R = 3g\cos 50°$

$F = \mu R$

$F = 0.571\ldots(3g\cos 50°)$

$F = 10.798\ldots N$

resolve parallel to plane (↙) with N2L

$F_{un} = ma$

$3g\sin 50° - F = 3a$

$3(9.8)\sin 50° - 10.798\ldots = 3a$

$a = 3.907\ldots\,ms^{-2}$

$u = 0\,ms^{-1}, a = 3.907\ldots ms^{-2}, s = 8, t = ?$

$s = ut + \frac{1}{2}at^2$

$8 = 0(t) + \frac{1}{2}(3.907\ldots)t^2$

$$t^2 = \frac{8(2)}{3.907\ldots}$$

$t = \pm 2.023\ldots$

$t > 0 \Rightarrow t = \underline{\underline{2.02s}}$ (3sf)

QUESTION NUMBER	
17.(a)	$\int x\sin 2x \, dx$ $\quad$ $\int uv' = uv - \int vu'$

$u \quad v'$

$u = x \quad v = -\frac{1}{2}\cos 2x$

$u' = 1 \quad v' = \sin 2x$

$$\int x\sin 2x \, dx = x\left(-\frac{1}{2}\cos 2x\right) - \int (1)\left(-\frac{1}{2}\cos 2x\right) dx$$

$$= -\frac{1}{2}x\cos 2x + \frac{1}{2}\int \cos 2x \, dx$$

$$= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + c$$

17.(b)

$V = \pi\int_a^b y^2 \, dx$ $\quad$ $y = \sqrt{x\sin 2x}$

$V = \pi\int_0^1 y^2 \, dx$ $\quad$ $y^2 = \left(\sqrt{x\sin 2x}\right)^2$

$y^2 = x\sin 2x$

$V = \pi\int_0^1 (x\sin 2x) \, dx$

$V = \pi\left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^1$

$V = \pi\left[\left(-\frac{1}{2}(1)\cos 2(1) + \frac{1}{4}\sin 2(1)\right) - \left(-\frac{1}{2}(0)\cos 2(0) + \frac{1}{4}\sin 2(0)\right)\right]$

$V = \pi\left[\left(-\frac{1}{2}\cos 2 + \frac{1}{4}\sin 2\right) - \left(0 + \frac{1}{4}\sin 0\right)\right]$

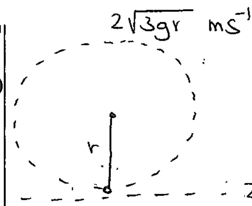
$V = \pi(0.435\ldots)$

$V = 1.3678\ldots$

$V = 1.37$ units^3 (3sf)

QUESTION NUMBER

18.(a)



maximum velocity of particle occurs when particle is at lowest point of circle

By conservation of energy,

At highest point At lowest point

KE + GPE = KE + GPE

$\frac{1}{2}m(2\sqrt{3gr})^2 + mg(2r) = \frac{1}{2}mv^2 + 0$

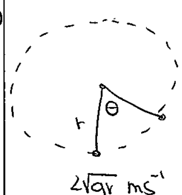
$\frac{1}{2}m(12gr) + 2mgr = \frac{1}{2}mv^2$

$8mgr = \frac{1}{2}mv^2$

$v^2 = 16gr$

$v = \sqrt{16gr} \Rightarrow v_{max} = 4\sqrt{gr}$ ms^{-1}

18.(b) (i)



By conservation of energy,

At lowest point At θ to downward vertical

KE + GPE = KE + GPE

$\frac{1}{2}m(2\sqrt{gr})^2 + 0 = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$

$\frac{1}{2}m(4gr) = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$

$4gr = v^2 + 2gr(1-\cos\theta)$

$v^2 = 2gr + 2gr\cos\theta$

$v^2 = 2gr(1+\cos\theta)$

$v = \sqrt{2gr(1+\cos\theta)}$

$h = r - r\cos\theta$

$h = r(1-\cos\theta)$

resolve radially (↖) with $F = \frac{mv^2}{r}$

$T - mg\cos\theta = \frac{mv^2}{r}$

$T - mg\cos\theta = \frac{m(2gr(1+\cos\theta))}{r}$

$T - mg\cos\theta = 2mg(1+\cos\theta)$

$T = mg(2+3\cos\theta)$

when tension in string is zero

$T = 0$

$mg(2+3\cos\theta) = 0$

$2 + 3\cos\theta = 0$

$3\cos\theta = -2$

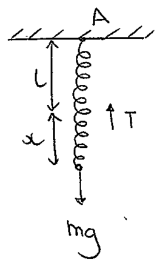
$\cos\theta = -\frac{2}{3}$ (as required)

18.(b)(ii)

If tension in string is zero, string is slack, particle is no longer ~~taut~~. constrained about vertical circle. Particle, ~~leaves the circle~~ ~~and~~ becomes a projectile.
no longer rotates about circle instead
moves freely under gravity

QUESTION NUMBER

19.(a)



$m = m$ kg
$L = L$ metres
$\lambda = 2mg$ newtons

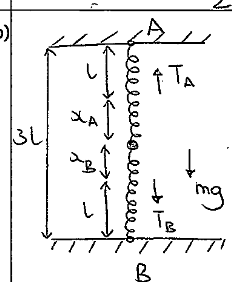
By Hookes Law: $T = \frac{\lambda x}{L}$

resolve (↑) at equilibrium

Let x = extension of the spring

$$T = mg$$
$$\frac{\lambda x}{L} = mg$$
$$\frac{2mgx}{L} = mg$$
$$2mgx = mgL$$
$$x = \frac{mgL}{2mg}$$
$$x = \frac{L}{2} \text{ metres} \quad \text{(as required)}$$

19.(b)



identical springs A and B

Let T_A = tension in spring A
T_B = tension in spring B
x_A = extension in spring A (original spring)
x_B = extension in spring B

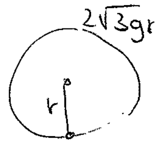
In equilibrium ⇒ forces are balanced
resolve (↑)

$3L - L - L = L$
$x_A + x_B = L$
$x_B = L - x_A$

$$T_A = T_B + mg$$
$$\frac{2mgx_A}{L} = \frac{2mgx_B}{L} + mg$$
$$\frac{2mgx_A}{L} = \frac{2mg(L - x_A)}{L} + mg$$
$$\frac{2mgx_A}{L} = 2mg - \frac{2mgx_A}{L} + mg$$
$$\frac{4mgx_A}{L} = 3mg$$
$$x_A = \frac{3L}{4} \text{ metres}$$

ENTER NUMBER OF QUESTION

ADDITIONAL SPACE FOR ANSWERS



$$\frac{1}{2}m(2\sqrt{3gr})^2 + 2mgr = \frac{1}{2}mv^2$$

$$6gr + 2mgr = \frac{1}{2}mv^2$$

$$v^2 = 16gr$$

$$= 4\sqrt{gr}$$