Candidate 1 evidence

QUESTION NUMBER	
6.(a)	$\cancel{\cos 3x \to 3\sin 3x \to 9\cos 3x \to +27\sin 3x \to 81\cos 3x}$ y $f(0) = 1$ $f'(0) = 0.$ $f''(0) = -9$ $f'''(0) = 0.$ $f^{IV}(0) = 81.$ $f(x) \simeq 1 - \frac{9x^2}{2} + \frac{81x^4}{24}$ $\simeq 1 - \frac{9}{2}x^2 + \frac{27}{8}x^4$ $f(x) = \cos 3x$ $f'(x) = 3\sin 3x$ $f''(x) = -9\cos 3x$ $f'''(x) = 27\sin 3x$ $f^{IV}(x) = 81\cos 3x$
6.(b)	$\cos^2 3x = (\cos 3x)(\cos 3x)$ $f(x) \simeq \left(1 - \frac{9}{2}x^2 + \frac{27}{8}x^4\right)\left(1 - \frac{9}{2}x^2 + \frac{27}{8}x^4\right)$ $= 1 - \frac{9}{2}x^2 + \frac{27}{8}x^4 - \frac{9}{2}x^2 + \frac{81}{4}x^4 + \frac{27}{8}x^4$ $= 1 - 9x^2 + 27x^4$

Candidate 2 evidence

QUESTION NUMBER

6.(a)

$f(x) = \cos 3x$

$f'(x) = -3\sin 3x$

$f''(x) = -9\cos 3x$

$f'''(x) = -27\sin 3x$

$f^{iv}(x) = -81\cos 3x$

$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!}$

$= 1 + 0 + \frac{-9x^2}{2} + 0 + \frac{-81x^4}{24}$

$= 1 + \frac{-9x^2}{2} + \frac{-81x^4}{24}$

6.(b)

$\cos^2 3x = \cos 2(3x) - 1$

$f(x) = \cos 2(3x) - 1$

$f'(x) = -6\sin 6x$

$f''(x) = -36\cos 6x$

$f'''(x) = -216\sin 6x$

$f^{iv}(x) = -1296\sin 6x$

$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!}$

$= 1 + 0 + \frac{-36x^2}{2} + 0 + \frac{-1296x^4}{24}$

$= 1 + -18x^2 + -54x^4$

Candidate 3 evidence

QUESTION NUMBER	
6.(a)	$f(x) = \cos 3x$ $\quad f(0) = \cos 3(0) = \cos 0 = 1$ $f'(x) = -3\sin 3x$ $\quad f'(0) = -3\sin 3(0) = -3\sin 0 = 0$ $f''(x) = -9\cos 3x$ $\quad f''(0) = -9\cos 3(0) = -9\cos 0 = -9$ $f'''(x) = 27\sin 3x$ $\quad f'''(0) = 27\sin 3(0) = 27\sin 0 = 0$ $f^{iv}(x) = 81\cos 3x$ $\quad f^{iv}(0) = 81\cos 3(0) = 81\cos 0 = 81$ $f(x) = 1 + 0x + \frac{-9x^2}{2!} + \frac{0x^3}{3!} + \frac{81x^4}{4!}$ $= 1 - \frac{9x^2}{2\times 1} + \frac{81x^4}{4\times 3\times 2\times 1}$ $= 1 - \frac{9x^2}{2} + \frac{81x^4}{24}$ $= 1 - \frac{9x^2}{2} + \frac{27x^4}{8}$
6.(b)	$f(x) = \cos^2 3x = (\cos 3x)^2$ $\quad f(0) = (\cos 3(0))^2 = (\cos 0)^2 = 1^2 \boxed{=1}$ $f'(x) = 2\cdot(\cos 3x)(-3\sin 3x)$ $= -6(\cos 3x)(\sin 3x)$ $\quad f'(0) = -6(\cos 3(0))(\sin 3(0)) = -6(\cos 0)(\sin 0)$ $\boxed{=0}$ $f''(x) = -6(\cos 3x)(3\cos 3x) + (-6)(\sin 3x)(-3\sin 3x)$ $= -18\cos^2 3x + 18\sin^2 3x = -18(\cos 3x)^2 + 18(\sin 3x)^2$ $\quad f''(0) = -18(\cos 3(0))^2 + 18(\sin 3(0))^2$ $\boxed{=-18}$ $f'''(x) = -18\cdot 2\cdot(\cos 3x)(-3\sin 3x) + 18\cdot 2\cdot(\sin 3x)(3\cos 3x)$ $= 108\cos 3x\sin 3x + 108\cos 3x\sin 3x$ $= 216\cos 3x\sin 3x$ $\quad f'''(0) = 216(\cos 3(0))(\sin 3(0))$ $\boxed{=0}$ $f^{iv}(x) = 216\cos 3x(3\cos 3x) + 216\sin 3x(-3\sin 3x)$ $= 648\cos^2 3x - 648\sin^2 3x$ $= 648(\cos 3x)^2 - 648(\sin 3x)^2$ $\quad f^{iv}(0) = 648(\cos 3(0))^2 - 648(\sin 3(0))^2$ $\boxed{=648}$ $f(x) = 1 + 0x - \frac{18x^2}{2!} + 0x^3 + \frac{648x^4}{4!}$ $f(x) = 1 - \frac{18x^2}{2\times 1} + \frac{648x^4}{4\times 3\times 2\times 1} = \underline{1 - 9x^2 + 27x^4}$

Candidate 4 evidence

7.(b)

$$\frac{d^2y}{dx^2} = \frac{d^2y}{dtdx} \times \frac{dt}{dx}$$

$$= \frac{4t\sec^2 t\tan t - 2\sec^2 t}{4t^2} \times \frac{1}{2t}$$

$$\frac{d^2y}{dx^2} = \underline{\underline{\frac{4t\sec^2 t\tan t - 2\sec^2 t}{8t^3}}}$$

Candidate 5 evidence

7.(b)

$$\frac{dx}{dt} = 2t \qquad \frac{d^2y}{dt^2} = 2 \cdot \sec t \tan t (\sec t) \quad (\sec t)^2$$

$$\frac{d^2x}{dt^2} = 2 \qquad = 2\sec^2 t \tan t$$

$$\frac{d^2y}{dx^2} = \frac{2\sec^2 t \tan t \cdot 2t - \sec^2 t \cdot 2}{(2t)^2}$$

$$= \frac{4t\sec^2 t \tan t - 2\sec^2 t}{4t^2}$$

$$= \frac{2t\sec^2 t \tan t - \sec^2 t}{2t^2}$$

Candidate 6 evidence

7.(b)	$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{\sec^2 t}{2t}\right) \times \frac{dt}{dx}$ $f = \sec^2 t \qquad g = 2t$ $f' = \sec^2 x \tan^2 x \qquad g' = 2$ $= \frac{2t\sec^2 x\tan^2 x - 2\sec^2 t}{(2t)^2} \times \frac{1}{2t}$ $= \frac{2t\sec^2 x\tan^2 x - 2\sec^2 t}{(2t)^3}$ $= \frac{2t\sec^2 x\tan^2 x - 2\sec^2 t}{8t^3}$ $= \frac{t\sec^2 x\tan x - \sec^2 t}{4x^3}$

Candidate 7 evidence

QUESTION NUMBER

12.

Aux Eq. $y = Ae^{3x} + Be^{5x}$

$m^2 - 8m + 15$

$(m-3)(m-5)$

$m = 3, m = 5$

$cx^2 + dx + e$

$\frac{dy}{dx} = 2cx + d$

$\frac{d^2y}{dx^2} = 2c$

$2c - 8(2cx + d) + 15(cx^2 + dx + e)$
$= 15x^2 - 31x + 40$

$2c - 16cx - 8d + 15cx^2 + 15dx + 15e$
$= 15x^2 - 31x + 40$

$\cancel{15cx^2}$

$15c = 15$

$c = 1$

$-16c + 15d = -31$

$-16 + 15d = -31$

$15d = -15$

$d = -1$

$\cancel{2c + 8d + 15e = 40}$

$\cancel{2 + 8 + 15e = 40}$

$\cancel{15e = 30}$

$\cancel{e = 2}$

$2c - 8d + 15e = 40$

$2 + 8 + 15e = 40$

$15e = 30$

$e = 2$

$\Rightarrow y = Ae^{3x} + Be^{5x} + x^2 - x + 2$

$\frac{dy}{dx} = 3Ae^{3x} + 5Be^{5x} + 2x - 1$

$13 = 3A + 5B - 1 \Rightarrow 14 = 3A + 5B$

$4 = A + B + 2 \Rightarrow 2 = A + B$

$6 = 3A + 3B$

$8 = 2B \quad A = -2$

$B = 4$

$\Rightarrow y = -2e^{3x} + 4e^{5x} + x^2 - x + 2$

Candidate 8 evidence

QUESTION NUMBER

12.

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 15x^2 - 31x + 40$$

$m^2 - 8m + 15 = 0$

$(m-5)(m-3) = 0$

$m-5=0 \quad m-3=0$

$m=5 \quad m=3$

$y = Ae^{5x} + Be^{3x}$

Try $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

Sub into equation:

$(2C)^2 - 8(2Cx + D) + 15(Cx^2 + Dx + E) = 15x^2 - 31x + 40$

$4C^2 - 16Cx - 8D + 15Cx^2 + 15Dx + 15E = 15x^2 - 31x + 40$

$4C^2 - 8D + 15E = 40$ ①

$-16Cx + 15Dx = -31x$ ②

$15Cx^2 = 15x^2$ ③

③

$15Cx^2 = 15x^2$

$\underline{C = 1}$

②

$-16Cx + 15Dx = -31$

$-16(1) + 15D = -31$

$15D = -15$

$\underline{D = -1}$

① $4C^2 - 8D + 15E = 40$

$4 \times 1^2 - 8(-1) + 15E = 40$

$4 + 8 + 15E = 40$

$15E = 28$

$E = 28/15$

QUESTION NUMBER

12. (cont)

$y = Ae^{5x} + Be^{3x} + x^2 - x + 28/15$

$\frac{dy}{dx} = 5Ae^{5x} + 3Be^{3x} + 2x - 1$

when $y=4,\ x=0$

$4 = Ae^0 + Be^0 + 0^2 - 0 + 28/15$

$4 = A + B + 28/15$

$A + B = \frac{32}{15}$ ①

when $x=0,\ dy/dx = 13$

$13 = 5Ae^0 + 3Be^0 + 2.0 - 1$

$5A + 3B = 14$ ② 14

$A + B = \frac{32}{15}$ ① $\times 3$

$5A + 3B = 14$ ②

$3A + 3B = \frac{32}{5}$ ③

$5A + 3B = 14$ ②

③ - ②

$-2A = -\frac{38}{5}$

$A = \frac{19}{5}$

~~when $A = 14/5$,~~

~~$B = \frac{32}{15} - \frac{14}{5}$~~

~~$= -2/3$~~

when $A = \frac{19}{5}$,

$B = \frac{32}{15} - \frac{19}{5}$

$= -5/3$

so $y = \frac{19}{5}e^{5x} - \frac{25}{3}e^{3x} + x^2 - x + \frac{28}{15}$

Candidate 9 evidence

QUESTION NUMBER

12.

$D^2 - 8D + 15 = 0$

$(D-5)(D-3) = 0$

$D = 5$ or $D = 3$

$y = Ae^{5x} + Be^{3x}$

$y = Ae^{5x} + Be^{3x}$

$\frac{dy}{dx} = 5Ae^{5x} + 3Be^{3x}$

$\frac{dy}{dx} = 13,\ x = 0 \qquad y = 4,\ x = 0$

$13 = 5A + 3B$ ① $\qquad 4 = A + B$ ②

② $\times -3 \qquad -12 = -3A - 3B$ ③

① + ③ $\qquad 1 = 2A$

$A = \frac{1}{2}$

Sub A into ②

$4 = \frac{1}{2} + B$

$B = \frac{7}{2}$

$y = \frac{1}{2}e^{5x} + \frac{7}{2}e^{3x}$

PI

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2xC + D$

$\frac{d^2y}{dx^2} = 2C$

$2C - 8(2xC + D) + 15(Cx^2 + Dx + E) = 15x^2 - 31x + 40$

$2C - 8(2xC + D) + 15Cx^2 + 15Dx + 15E = 15x^2 - 31x + 40$

$2C - 16xC - 8D + 15Cx^2$

Equate x^2

$15Cx^2 = 15x^2$

$15C = 15$

$C = 1$

Equate x

$-16C + 15D = -31$

$-16 + 15D = -31$

$15D = -15$

$D = -1$

Equate k

$2C - 8D + 15E = 40$

$2 + 8 + 15E = 40$

$15E = 30$

$E = 2$

PI $\qquad x^2 - x + 2$

$GS = PI + CS$

$= \frac{1}{2}e^{5x} + \frac{7}{2}e^{3x} + x^2 - x + 2$

Candidate 10 evidence

14.

$$\frac{dy}{dx} - \frac{2}{x}y = x^2\sec^2 3x$$

Integrating factor $= e^{\int -\frac{2}{x}}$

$$\int -\frac{2}{x} = -2\int \frac{1}{x}$$
$$= -2\ln x$$
$$e^{-2\ln x} = x^{-2}$$
$$x^{-2} = \frac{1}{x^2}$$

$$y \cdot x^{-2} = \cancel{\int x^{-2} \cdot \sec^2 3x}$$
$$= \int x^{-2} \cdot x^2 \sec^2 3x$$
$$= \int \sec^2 3x$$
$$= \frac{1}{3}\tan(3x) + c$$
$$y = \cancel{\frac{\frac{1}{3}\tan(3x)}{}}$$
$$= \frac{x^2}{3}\tan(3x) + x^2 c$$

Candidate 11 evidence

14.

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 \sec^2 3x$$

$$P(x) = -\frac{2}{x} \qquad \text{I.F} = e^{\int -2/x\,dx} = e^{-2\int 1/x\,dx} = e^{-2\ln x}$$

$$= e^{\ln x^{-2}}$$

$$= \cancel{e}-x^2$$

$$\frac{dy}{dx}\cdot -x^2 - \frac{2}{x}y\cdot -x^2 = x^2\sec^2 3x \cdot x^2$$

$$-x^2 y = -x^2\left(x^2 \sec^2 3x\right)$$

$$-x^2 y = -x^4 \sec^2 3x$$

$$-x^2 y = \int -x^4 \sec^2 3x \quad dx$$

$$= -\frac{1}{5}x^5 \cdot \frac{1}{3}\tan(3x) + c$$

$$= -\frac{1}{15}x^5 \tan 3x + c$$

$$y = \frac{x^5 \tan 3x + c}{15x^2}$$

Candidate 12 evidence

14.

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 \sec 3x$$

~~dx~~ integrating factor

$$= e^{\int -\frac{2}{x}}$$

$$= e^{-2\int \frac{1}{x}}$$

$$= e^{-2\ln x}$$

$$= e^{\ln x^{-2}}$$

$$= x^{-2}$$

$$= \frac{1}{x^2}$$

$$\frac{d}{dx}\left(\frac{y}{x^2}\right) = \sec 3x$$

$$\frac{y}{x^2} = \int \sec 3x \, dx$$

$$\frac{y}{x^2} = \frac{1}{3}\sec 3x \tan 3x + C$$

$$y = \frac{x^2}{3}\sec 3x \tan 3x + Cx^2$$

Candidate 13 evidence

QUESTION NUMBER

15.

$$\sum_{r=1}^{n} \frac{1}{(2r+1)(2r-1)} = \frac{n}{2n+1}$$

let $n=1$

LHS

$$\sum_{r=1}^{1} \frac{1}{(2r+1)(2r-1)}$$

RHS

$$\frac{1}{3}$$

$$= \frac{1}{(3)(1)}$$

$$= \frac{1}{3}$$

as LHS = RHS

initially true for

$n=1$

let $n=k$

LHS

$$\sum_{r=1}^{k} \frac{1}{(2r+1)(2r-1)}$$

RHS

$$\frac{k}{2k+1}$$

let $n=k+1$

LHS

$$\sum_{r=1}^{k+1} \frac{1}{(2r+1)(2r-1)}$$

RHS

$$\frac{k+1}{2k+3}$$

$$\sum_{r=1}^{k} \frac{1}{(2r+1)(2r-1)} + \frac{1}{(2k+3)(2k+1)}$$

assume true for $n=k$

so

$$\frac{k}{2k+1} + \frac{1}{(2k+3)(2k+1)}$$

$$\frac{k(2k+3)}{(2k+1)(2k+3)} + \frac{1}{(2k+3)(2k+1)}$$

QUESTION NUMBER

15. (cont)

$$\frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$\frac{2k^2+3k+1}{(2k+1)(2k+3)}$$

$$\frac{(k+1)(2k+1)}{(2k+1)(2k+3)}$$

$$\frac{k+1}{2k+3}$$

hence proven true by induction for $n=k+1$ so also ~~the~~ proven true for $n=k$. Shown true for $n=1$. hence true by induction for all posotive integers n

Candidate 14 evidence

QUESTION NUMBER

15.

$$\sum_{r=1}^{n} \frac{1}{(2r+1)(2r-1)} = \frac{n}{2n+1}$$

Consider $n=1$

$LHS = \frac{1}{(2+1)(2-1)}$ $= \frac{1}{3}$ $RHS = \frac{1}{2+1}$ $= \frac{1}{3}$ $LHS = RHS$

$\therefore$ true for $n=1$

Assume true for $n=k$

$$\sum_{r=1}^{k} \frac{1}{(2r+1)(2r-1)} = \frac{k}{2k+1}$$

Aim

$$\sum_{r=1}^{k+1} \frac{1}{(2r+1)(2r-1)} = \frac{k+1}{2k+3} \quad \cancel{\frac{k+1}{(2k+3)(2k+1)}}$$

Consider $n=k+1$

$$\sum_{r=1}^{k+1} = \frac{1}{(2r+1)(2r-1)} = \sum_{r=1}^{k} \frac{1}{(2r+1)(2r-1)} + (k+1)^{th} \text{ term}$$

$$= \frac{k}{2k+1} + \frac{1}{(2(k+1)+1)(2(k+1)-1)}$$

$$= \frac{k}{2k+1} + \frac{1}{(2k+3)(2k+1)}$$

$(k(2k+3)+1)$
$=2k+5k+1$
$=(2k+1)(k+1)$

$$= \frac{k(2k+3)(2k+1) + (2k+1)}{(2k+1)^2(2k+3)}$$

$$= \frac{(2k+1)(k(2k+3)+1)}{(2k+1)^2(2k+3)}$$

$$= \frac{(2k+1)(2k+1)(k+1)}{(2k+1)^2(2k+3)}$$

$$= \frac{k+1}{2k+3}$$

$\therefore$ true for $n=k+1$

If true for $n=k$, then true for $n=k+1$. Also true for $n=1$ $\therefore$ by proof by induction true for all positive integers n.

Candidate 15 evidence

QUESTION NUMBER 15.

$$\sum_{r=1}^{n} \frac{1}{(2r+1)(2r-1)} = \sum_{r=1}^{n} \frac{1}{4r^2-1}$$

$n=1$

$$\sum_{r=1}^{1} \frac{1}{4(1)^2-1} = \frac{1}{3} \qquad \frac{1}{2(1)+1} = \frac{1}{3}$$

LHS=RHS ∴ true for $n=1$.

QUESTION NUMBER 15. (cont)

$n=k$

$$\sum_{r=1}^{k} \frac{1}{4r^2-1} = \frac{k}{2k+1}$$

$n=k+1$

$$\sum_{r=1}^{k+1} = \sum_{r=k+1}^{k+1} + \sum_{r=1}^{k}$$

$$\frac{1}{4(k+1)^2-1} + \frac{k}{2k+1} \cancel{= \frac{k+1}{2(k+1)+1} = \frac{k+1}{2k+3}}$$

$$\frac{1}{4k^2+8k+4-1} + \frac{k}{2k+1} = \frac{1}{(2k+3)(2k+1)} + \frac{k}{2k+1}$$

$$= \frac{1+k(2k+3)}{(2k+3)(2k+1)} = \frac{2k^2+3k+1}{(2k+3)(2k+1)} = \frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$$

$$= \frac{k+1}{2k+3}$$

RHS

$$\frac{k+1}{2(k+1)+1} = \frac{k+1}{2k+3}$$

LHS = RHS for $n=k+1$, assuming LHS=RHS for $n=k$, and ~~given~~ proven true for $n=1$, it can be proven by induction that the statement is true $\forall n$.

Candidate 16 evidence

QUESTION NUMBER

17.

$$\frac{dV}{dh} = \frac{3}{5}h^2$$

$$\frac{dV}{dt} = 6 - \frac{1}{10\sqrt{h}} = 6 - \frac{1}{10}(h)^{1/2}$$

$$= -\frac{1}{20}(h)^{-1/2} = -\frac{1}{20\sqrt{h}}$$

$$\cancel{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}}$$

$$\cancel{= -\frac{1}{20\sqrt{h}} \times \frac{5}{3h^2}}$$

$$\cancel{= -\frac{1}{12\sqrt{h^3}}}$$

$$\frac{dt}{dh} = \frac{dV}{dh} \div \frac{dV}{dt}$$

$$= \frac{3}{5}h^2 \times -\frac{20\sqrt{h}}{1}$$

$$= -12\sqrt{h^5}$$

When $h = 400$:

$$= -12\sqrt{(400)^5}$$

$$= -3.84 \times 10^{-7}$$

Candidate 17 evidence

QUESTION NUMBER

17.

$V = \frac{1}{5}h^3$ $\qquad \frac{dV}{dt} = 6 \qquad \frac{dV}{dt} = -\frac{1}{10}\sqrt{h}$

$\frac{dV}{dh} = \frac{3}{5}h^2 = \frac{3h^2}{5} \qquad \frac{dV}{dt} = 6-2 \qquad \frac{dV}{dt} = -\frac{\sqrt{400}}{10} = -2$

$= 3 \text{ cm}^3\text{s}^{-1}$

$\cancel{\frac{dh}{dt} =}$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{5}{3h^2} \times 3$

$= \frac{15}{3h^2} = \frac{5}{h^2}$

<u>sub h = 400</u>

$\frac{dh}{dt} = \frac{5}{400^2}$

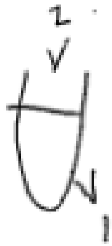
$= 3.125 \times 10^{-5} \text{ cm}^3\text{ s}^{-1}$.

Candidate 18 evidence

QUESTION NUMBER

17.

$V = \frac{1}{5}h^3 \quad \frac{dv}{dh} = \frac{3}{5}h^2$



$\frac{dv}{dt} = 6 \quad \frac{dv}{dt} = -\frac{1}{10}\sqrt{h}$

$2 - 1 = 1$

$\frac{dv}{dt} = 6 - \frac{1}{10}\sqrt{h}$

$\frac{dh}{dt} = \frac{dv}{dh} \times \frac{dh}{dt} = \frac{3}{5}h^2 \times \frac{1}{\left(6 - \frac{1}{10}\sqrt{h}\right)}$

$= \frac{3h^2}{5\left(6 - \frac{1}{10}\sqrt{h}\right)}$

$h = 400$

$\cancel{\frac{dv}{dt}} \quad = \frac{480000}{20}$

$\frac{dh}{dt} = 24000$ cm/second